Quiz 2

October 19, 2018

Determine the truth or falsity of each of the following statements. Be sure to explain your answers.

1. If A is invertible and AB is invertible, then B is invertible.

Solution. This is true. If A and AB are invertible, then A^{-1} is invertible and

$$B = IB = (A^{-1}A)B = A^{-1}(AB).$$

Thus, as the product of invertible matrices is invertible, B is invertible.

2. If A is not invertible and B is not invertible, then AB is not invertible.

Solution. This is false. Let

$$A = \begin{bmatrix} 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Then neither A nor B is invertible, but

$$AB = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = [2]$$

is invertible.

3. There exists a vector

$$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \in \mathbb{R}^3$$

such that $[T]$ is invertible, where T is the linear function $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by the formula

$$T(\vec{v}) = \vec{a} \times \vec{v}, \qquad \vec{v} \in \mathbb{R}^3.$$

Solution. This is false. Choose a nonzero vector $\vec{x}$ in $\mathbb{R}^3$ that is colinear with $\vec{a}$. Then

$$[T]\vec{x} = T(\vec{x}) = \vec{a} \times \vec{x} = 0.$$

But then $[T]$ cannot be invertible as it cannot have a left inverse: if $B[T] = I$, then

$$\vec{x} = I\vec{x} = B[T]\vec{x} = 0,$$

contradicting the fact that $\vec{x}$ was chosen to be a nonzero vector.